# Exercise 10

## I. Consequences of the grading

Depending on whether $A$, $B$ are in $g_0$ or $g_1$, the generalized Lie product either satisfies

$$[A,B] = -[B,A], \text{which we denote as } [A,B]_- \text{or} \tag{1}$$
$$[A,B] = [B,A], \text{which we denote as } [A,B]_+. \tag{2}$$

1. Which cases result in $[A,B]_-$ and which in $[A,B]_+$?
2. Express the generalized Jacobi identity in terms of the $[\ ,\ ]_-$ and $[\ ,\ ]_+$ for the different combinations of $A$, $B$, $C$ in $g_0$ or $g_1$.

## II. The commutation relations of the generators of $A(1|0)$

When represented as matrices, the elements of a superalgebra are block matrices

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}. \tag{3}$$

If $M \in g_0$, it has the form

$$M = \begin{pmatrix} A & 0 \\ 0 & D \end{pmatrix}, \tag{4}$$

while if $M \in g_1$, it has the form

$$M = \begin{pmatrix} 0 & B \\ C & 0 \end{pmatrix}. \tag{5}$$

In the case of $A(1|0)$, the generators are $3 \times 3$ matrices with $A$ being a $2 \times 2$ matrix and $D$ a $1 \times 1$ matrix. Let $E_{pq}$ be the $3 \times 3$ matrix with entry $(p,q) = 1$ and all others zero. The generators of the Cartan subalgebra are given by $H_1^1 = E_{11} - E_{22}$ and $C = -E_{11} - E_{22} - 2E_{33}$. $E_{12}$, $E_{21}$, $E_{13}$, $E_{31}$, $E_{23}$ and $E_{32}$ generate the root subspaces.

1. Give the commutation relations of $A(1|0)$.
2. Which of the generators $E_{pq}$ belong to even roots and which of them belong to odd roots?

## III. The Killing form of $A(1|0)$

As in the simple case, we define $\mathrm{ad}(X)Y = [X,Y]$, but now using the supercommutator. The Killing form is defined as

$$K(X,Y) = \mathrm{str}(\mathrm{ad}X \circ \mathrm{ad}Y), \quad X,Y \in g_s, \tag{6}$$

where str is the **supertrace**, defined as

$$\mathrm{str}M = \mathrm{Tr}\,A - \mathrm{Tr}\,D \tag{7}$$

for the block matrix $M$. Take the basis elements of $A(1|0)$ in the order

$$C,\ H_1^1,\ E_{12},\ E_{21},\ E_{13},\ E_{31},\ E_{23},\ E_{32}. \tag{8}$$

1. Calculate $\mathrm{ad}C$ and $\mathrm{ad}H_1^1$.
2. Give $K(C,C)$, $K(H_1^1,H_1^1)$ and $K(C,H_1^1)$.